A Hybrid Method Realizing Shock Filter by Cellular Neural Network

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Abstract—Shock filter is an important type of the filters in image processing. In recent years, research on shock filters has been an interested topic in signal and image processing. In this paper we propose a method to realize the shock filter by CNN (Cellular Neural Network). The method offers fast processing time in comparison with that on the serial digital computers and can be used in many real-time image processing systems. We focus on development of the shock filter algorithm realizing edge enhancement and de-noising on the Bi-I CNN visual computer. Besides presenting the algorithm, many experiments on Bi-I are given for demonstration.

I. INTRODUCTION

A. The Classical Shock Filters

The basic principle of the shock filters are shown in Figure 1

![Figure 1. a) Blur edge input b) Shock filter output](image)

The classical shock filter can be described by the following partial differential equation [3]:

\[ I_t = -|F_x|F(I_{xx}) \] (1)

where \( F \) function should satisfy \( F(0) = 0, F(s) \text{ sign}(s) \geq 0 \).

Note that the above equation and all other evolutionary equations in this paper have initial conditions \( I(x, 0) = I_0(x) \) and Neumann boundary conditions.

Choosing \( F(s) = \text{sign}(s) \) gives the classical shock filter equation:

\[ I_t = -\text{sign}(I_{\eta \eta})|F_x| \] (2)

In the 2D case the shock filter equation is commonly generalized to:

\[ I_t = -\text{sign}(I_{\eta \eta})|\nabla I| \] (3)

where \( \eta \) is the direction of the gradient.

The weakness of classical shock filter is extremely sensitive to noise, that is any noise in the blurred signal will be enhanced.

B. The Advanced Shock Filters

Alvarez and Mazorra (1994) firstly combined shock and diffusion in their work [4], [5] as following equation:

\[ I_t = -\text{sign}(G_{\sigma} \ast I_{\eta \eta})|\nabla I| + c \varepsilon \] (4)

where \( c \) is a positive constant and \( \varepsilon \) is the perpendicular direction to the gradient \( \nabla I \). The first term is shock, and the second is linear diffusion term. This model better performs enhancing and denoising along of edges.

Kornprobst et al (1997) proposed a process evolution of the shock filter equation, which is a combination of many components that execute enhancing edges and denoising [6]:

\[ I_t = -\alpha_1(1-h\tau)\text{sign}(G_{\sigma} \ast I_{\eta \eta})|\nabla I| + \alpha_2(h\sigma_{\eta \eta} + I_{\kappa}) \] (5)

where \( h\tau = h\tau(I|G_{\sigma} \ast \nabla I|) = 1 \) if \( |G_{\sigma} \ast \nabla I| < \tau \) and = 0 otherwise.

If the coupling coefficients \( \alpha_1, \alpha_2, \alpha_3 \) are appropriately chose then the enhancing edges and denoising are impeccably performed.

Coulon and Arridge (2000) presented shock filters based on a probabilistic function [7]:

\[ I_t = -(1-c)^a \text{sign}(G_{\sigma} \ast I_{\eta \eta})|\nabla I| + \text{div}(c\nabla I) \] (6)

where \( c = \exp (-|G_{\sigma} \ast \nabla I|^2/\kappa) \). This equation performs a process of classical anisotropic diffusion for low edge values, and a shock filter for high values, with an intermediate transition zone. The weighting by \((1-c)^a\) prevents shocks to be developed in the presence of noise inside objects. Only edges are enhanced. The control is stronger when \( a \) is higher.

Gilboa et al. (2001) used complex components on diffusion equations for image processing [8], [9], and [10]. After processing, the origin image is changed into two images: the real part is reduced noise image and the imaginary part serves as a robust edge-detector.

The PDE equation for nonlinear complex diffusion approach has the form:
\[
\frac{\partial}{\partial t} I = \nabla \cdot (d(\text{Im}(I)) \nabla I)
\]  
(7)

The \text{Im}(I) in (7) is the imaginary part of \(I\) and the diffusivity coefficient \(d(\text{Im}(I))\) is defined as
\[
d(\text{Im}(I)) = \exp(i\theta) \frac{1 + \left(\text{Im}(I)/k\right)}{1 + \left(\text{Im}(I)/k\right)^2}
\]
(8)

where \(k\) is a threshold parameter and \(\theta \in (-\pi/2;\pi/2)\) is the phase angle. Gilboa showed that the derived nonlinear complex processes are good only for small theta values [10]. As \(\theta \to 0\) the imaginary part can be considered as a smoothed second derivative of the initial signal. Generalizing the solution to any dimensions in Cartesian coordinates, we show that [10]
\[
\lim_{\theta \to 0} \frac{\text{Im}(I)}{\theta} = t\Delta g_\theta \ast I_0
\]
(9)

where \(\text{Im}(\cdot)\) is imaginary value and \(\bar{\sigma} = \lim_{\theta \to 0} \sigma = \sqrt{2t}\).  

Substitute \(F(s) = 2\pi \arctan(as)\) into (3), yields
\[
I_i = -\frac{2}{\pi} \arctan(al_{xx})|I| + \lambda I_{xx}
\]
(10)

where \(a\) is a parameter that controls the sharpness of the slope near zero. From (9) and (10) we get:
\[
I_i = -\frac{2}{\pi} \arctan(al_{m}(\frac{I}{\theta}))|I| + \lambda I_{xx} \quad \text{with } \lambda = re^{i\theta}.
\]
(11)

Similarly the complex shock filter for 2D image has the form:
\[
I_i = -\frac{2}{\pi} \arctan(al_{m}(\frac{I}{\theta}))|I| + \lambda I_{xx} + \tilde{\lambda} I_{ss}
\]
(12)

where \(\tilde{\lambda}\) is a real scalar.

In order to perform (12) on PC this equation is approximated separated into:
\[
D_l f_{i,j} = \arctan(\text{Im}(f_{i,j})/\theta) \left(\frac{\Delta x f_{i,j} + \Delta y f_{i,j}}{2} + \frac{\Delta x f_{i,j} + \Delta y f_{i,j}}{2} + \tilde{\lambda} D_l f_{i,j}ight)
\]
(13)

where \(D_{l}(x,y)\) is a symmetric first order approximation in the \(x\) or \(y\) direction defined by the minmod function, \(D_{\eta \eta}\) approximates derivative in the gradient direction, and \(D_{\sigma \sigma}\) approximates derivative in the direction perpendicular to the gradient. The terms \(\Delta x f_{i,j}\) and \(\Delta y f_{i,j}\) are computed by minmod function (see section 3 in this paper). Recently, Jeny Rajan, K. Kannan, and M.R. Kaimal in their work [14] proclaimed that the complex nonlinear diffusion could perform even with higher values. In this case the diffusion is quickly performed.

Chen Guan-nan, Yang Kun-tao, Chen Rong, Xie Zhi-ming

Zhi-ming (2008) [13] proposed some improvement in shock filter to rise the denoising and reducing time of computation. Firstly they replaced the “soft” sign of the signal shock function with a hyperbolic tangent function \(F(s) = \text{th}(a(s))\), where \(a\) controls the slope of hyperbolic tangent. Secondly, a fidelity term \(\lambda_2(I_0 - I)\) is added for preserving geometric information of the original image. Their shock filter is:
\[
I_i = \text{th}(\lambda_2(I_0 - I)) \nabla I + \lambda_1 I_{\eta \eta} + \lambda_4 (I - I_i)\]
(14)

Because the convolutions of the sign(.) function are avoided so the time of computing is greatly reduced.

II. CELLULAR NEURAL NETWORK

Cellular Neural Networks [1, 2] are lattices of cells that are locally connected analogical processors. CNNs can be implemented in VLSI or FPGA technology and perfectly suitable for analog image processing. The operation of a cell at location \((i,j)\) is described by the following dimensionless equations:
\[
\frac{d x_{i,j}}{d t} = -x_{i,j} + A \otimes y_{i,j} + B \otimes u_{i,j} + I
\]
(15)

\[
y_{i,j}(x) = \frac{1}{2}(1(x_{i,j} + 1) - 1(x_{i,j} - 1))
\]
(16)

where \(\otimes\) denotes a two-dimensional discrete spatial convolution such that \(A \otimes y_{i,j} = \sum_{k,l \in N(i,j)} A_{i,j} y_{i+k,j+l} \) for \(k, l\) in the neighborhood \(N(i,j)\) of cell \((i,j)\). When \(r = 1\), the number of neighboring cells is \(8\). Matrices \(A\) and \(B\) are respectively the so-called feedback and feed forward weighting matrices, and \(z_{ij}\) is the cell bias, \(u_{i,j}, x_{i,j}, y_{i,j}\) are the input, internal state and output of a cell \((i,j)\), respectively. The same set of parameters \(A\), \(B\) and \(z\), also known as the cloning template, is repeated periodically for each cell over the whole network, which implies a reduced set of at most 19 control parameters, but nevertheless a large number of possible processing operations. The template sets the state of a CNN circuit after a transient process. When an array is loaded into the input of CNN, we obtain an output array after approximately a few microseconds. The outcome of output array depends solely on the templates, the input array \(u\), and the initial state of the internal array \(x\).

Architecture of CNN universal machine is composed of CNN programmable arrays, sensor arrays and DSP devices. For example the CNN-UM Bi-I V2 has architecture as shown in Figure 3.

The image can be captured from sensor CMOS that have high resolution or from sensor-processor CNN ACE16K. Structure of CNN array in this chip is the same as retina of mammalian animal. Number of processors in ACE16K is 16384 cells. The maximum of signal array that is processed parallel in this chip is 128x128. When the image resolution is bigger than the size of the CNN chip, the Tilling method [11],

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[11], [12]) can be used to divide the image into blocks so that they can be parallel processed using CNN. DSP devices are served for arithmetic operations. The processor ETRAX100 is used in communication between Bi-I and PC via Ethernet, RS 232 or USB channels. The newest series of family Bi-I is V3, its architecture is also like of V2. Therefore Bi-I is a combine of true parallel processing on CNN and serial processing on DSP.

![Diagram](ThB7.1)

Fig. 2. The 2D two layer CNN for solving complex linear diffusion equation.

### III. Realizing Shock Filter by Bi-I CNN Visual Computer

We start with the shock filter model (4) proposed by Alvarez and Mazorra’s [4], [5]. Other models (5), (6), (13), (14) also can be realized on Bi-I CNN visual machine. The shock filter models are very noise sensitive, so usually before realizing shock filter, to reduce noise in the original image a spatial Gaussian convolution filter \(G_x \star I_{xy}\) is performed. The spatial convolution is very time consuming computation on the traditional serial digital computer. However this is not a problem with Bi-I CNN visual machine due to parallel processing capability of CNN chip in Bi-I. We propose a method to realize the shock filter by Bi-I. With Bi-I we can do many image parallel processing functions of shock filter in only few microseconds, therefore total computing time is very fast in comparison with the computation time on the traditional digital computer. The proposed algorithm is given in detail as follow.

\[
I_{m}(i,j) = I(i,j)-I(i-1,j) \text{ and } I_{py} = I(i,j+1)-I(i,j)
\]

The shock filter (4) can be written in the form:

\[
I_x = -\text{sign}(G_x \star I_{yy}) \sqrt{\left|D_x\right|^2 + \left|D_y\right|^2 + cI_x} \tag{17}
\]

where \(D_x = \text{minmod} (I_{mx}, I_{px})\); \(D_y = \text{minmod} (I_{my}, I_{py})\) with:

- \(\text{minmod} (I_{mx}, I_{px}) = \min(I_{mx}, I_{px})\) if \((I_{mx}, I_{px}) > 0\)
- \(\text{minmod} (I_{my}, I_{py}) = 0\) if \((I_{my}, I_{py}) \leq 0\)

and

- \(\text{minmod} (I_{my}, I_{py}) = \min(I_{my}, I_{py})\) if \((I_{my}, I_{py}) > 0\)
- \(\text{minmod} (I_{my}, I_{py}) = 0\) if \((I_{my}, I_{py}) \leq 0\)

If we denote a pixel \(I_{ij}\) and its neighbors as follow

| \(I(i-1, j-1)\) | \(I(i-1, j)\) | \(I(i-1, j+1)\) |
| \(I(i, j-1)\) | \(I(i, j)\) | \(I(i, j+1)\) |
| \(I(i+1, j)\) | \(I(i+1, j-1)\) | \(I(i+1, j+1)\) |

then equation (4) can be discretized as

\[
(I_x)_{i,j} = (I_{i+1,j} - I_{i-1,j})/2h
\tag{18}
\]

\[
(I_y)_{i,j} = (I_{i,j+1} - I_{i,j-1})/2h
\tag{19}
\]

\[
(I_{xx})_{i,j} = (I_{i+1,j} - 2I_{i,j} + I_{i-1,j})/2h^2
\tag{20}
\]

\[
(I_{yy})_{i,j} = (I_{i,j+1} - 2I_{i,j} + I_{i,j-1})/2h^2
\tag{21}
\]

\[
(I_{xy})_{i,j} = (I_{i+1,j+1} - I_{i+1,j-1} - I_{i-1,j+1} + I_{i-1,j-1})/4h^2
\tag{22}
\]

\[
(I_{xy})_{i,j} = (I_x^2 I_{xx} + 2I_x I_y I_{xy} + I_y^2 I_{yy})/(I_x^2 + I_y^2)
\tag{23}
\]

\[
(I_{xx})_{i,j} = (I_x^2 I_{xx} - 2I_x I_y I_{xy} + I_y^2 I_{yy})/(I_x^2 + I_y^2)
\tag{24}
\]

Follow chart of the proposed method is given in Figure 4. Here \(I \in \mathbb{R}\) and \(I(x, 0) = I_0 \in \mathbb{R}\). In the each step processing by CNN, the initial image is the original image (in the first step) or image of the previous step (in the midway steps). The bounded condition is Neumann condition.
EXPERIMENTS

We have done several experiments on Bi-I CNN visual machine to demonstrate the correct operation and also to test the processing time of the proposed complex shock filter algorithm. We present here the results of two experiments.

In the first experiment shock filter was tested on a color image "ha_long.bmp" size of 128x100 that is stored in a hard disk. The results are given in Figure 5.

![Figure 5](image-url)

Figure 5. a) the origin image "ha_long.bmp" size of is 128x100 b) the origin image with noise NSR = 5dB c) the noise image after 5 iterations of shock filter, time total of processing equals 45 milliseconds. d) the noise image after 20 iterations of shock filter, time total of processing equals 200 milliseconds.

In the second experiment the image captured by camera on Bi-I reaches to speed of 1000fps and the shock filter is also realized successfully with this speed.

If size of the images is bigger than 128x128 then Tilling method can be used. In this case the speed of processing is strongly downed because the steps of 5, 6, 7, 11, 12, 14, and 15 are computed serially with bigger arrays and with also time of the Tilling. To overcome these computations in steps 5, 6, 7, 11, 12, 14, 15 must approximated by a way to be able computed parallel by CNN.
V. CONCLUSION

The shock filters are used for image debluring to enhancing the difference of intensity at the edges in image processing. The classical shock filter is very sensitivity to noise. The advances of shock filter by coupling shock and diffusion, adding factored coefficients, performing combination many components in shock filter equation, using complex shock filter one has to get many good results in image processing. However the computations on serial digital computer are very time consuming and may not applicable in real-time applications. To speed up we have proposed a hybrid parallel/serial computing algorithm for shock filter realizable on Bi-I CNN visual machine. Using appropriate templates most computation of the shock filter can be done parallel by CNN chip and the rest is done by DSP of the BI-I. The correctness and performances of the proposed algorithms were demonstrated by experiments on the Bi-I CNN visual machine at the Faculty of the Information Technology, Thai Nguyen University.

By performing the hybrid method on the CNN universal machine like Bi-I one can speed up shock filter and other computation in image and signal processing.

REFERENCES


